
بسم الله الرحمن الرحیم

Topological

Theory

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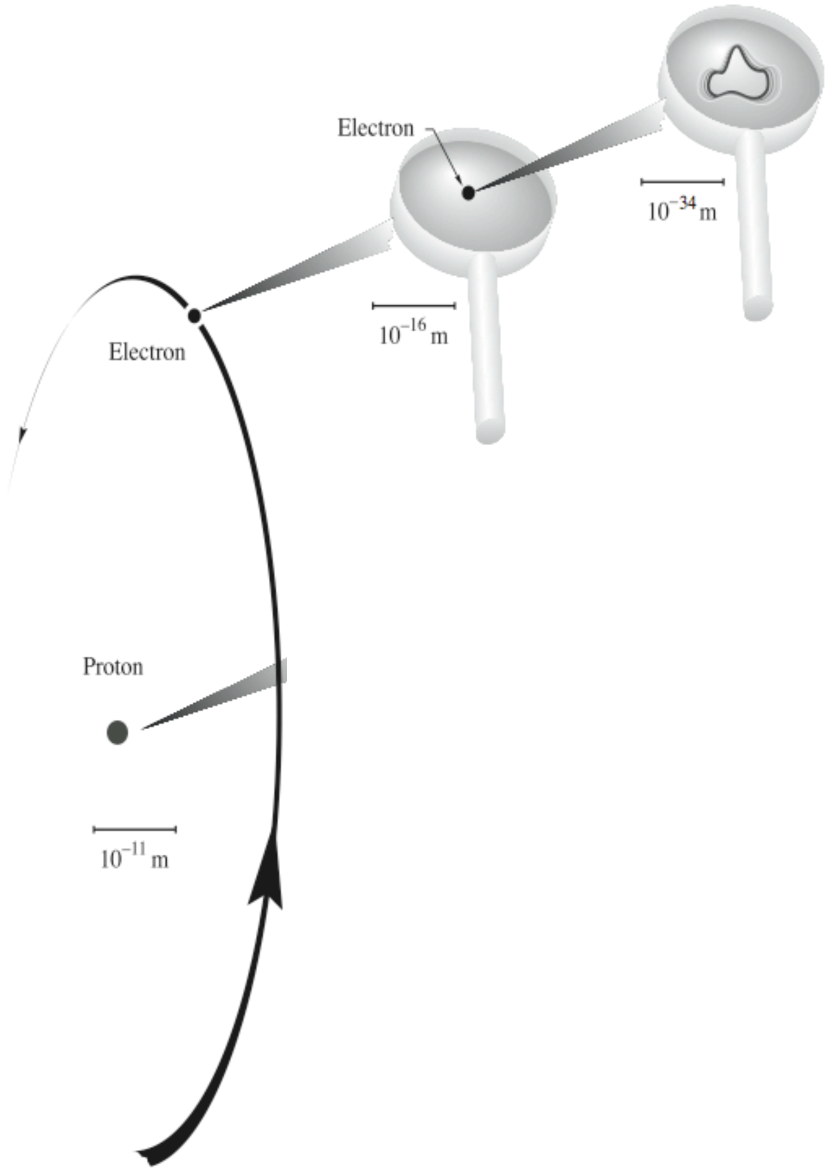
- <u>The Basic Argument</u>

String theory is a physics theory that the universe is composed of vibrating filaments of energy, expressed in precise mathematical language. These strings of energy represent the most fundamental aspect of nature.



The hydrogen atom consists of an electron and a proton, each of which, according to string theory, ultimately consists of vibrating strings.

(Courtesy of Terry Anderson and Lance Dixon, Stanford Linear Accelerator Center)

Supersymmetry

Nature of space-time at Planck scale (10^{-34} m) is non-commutative because there are no points.

The goal is to construct a geometry that is non-commutative but has the Riemann-Einstein geometry as a limit.

Salam and Strathdee among others suggested that the classical manifold, should be replaced by a manifold $\mathscr{M}$, such that it admits additional set of anti-commuting (Grassman) coordinates.

Supergeometry

On our supermanifold $\mathscr{M}$, the local coordinates are given by

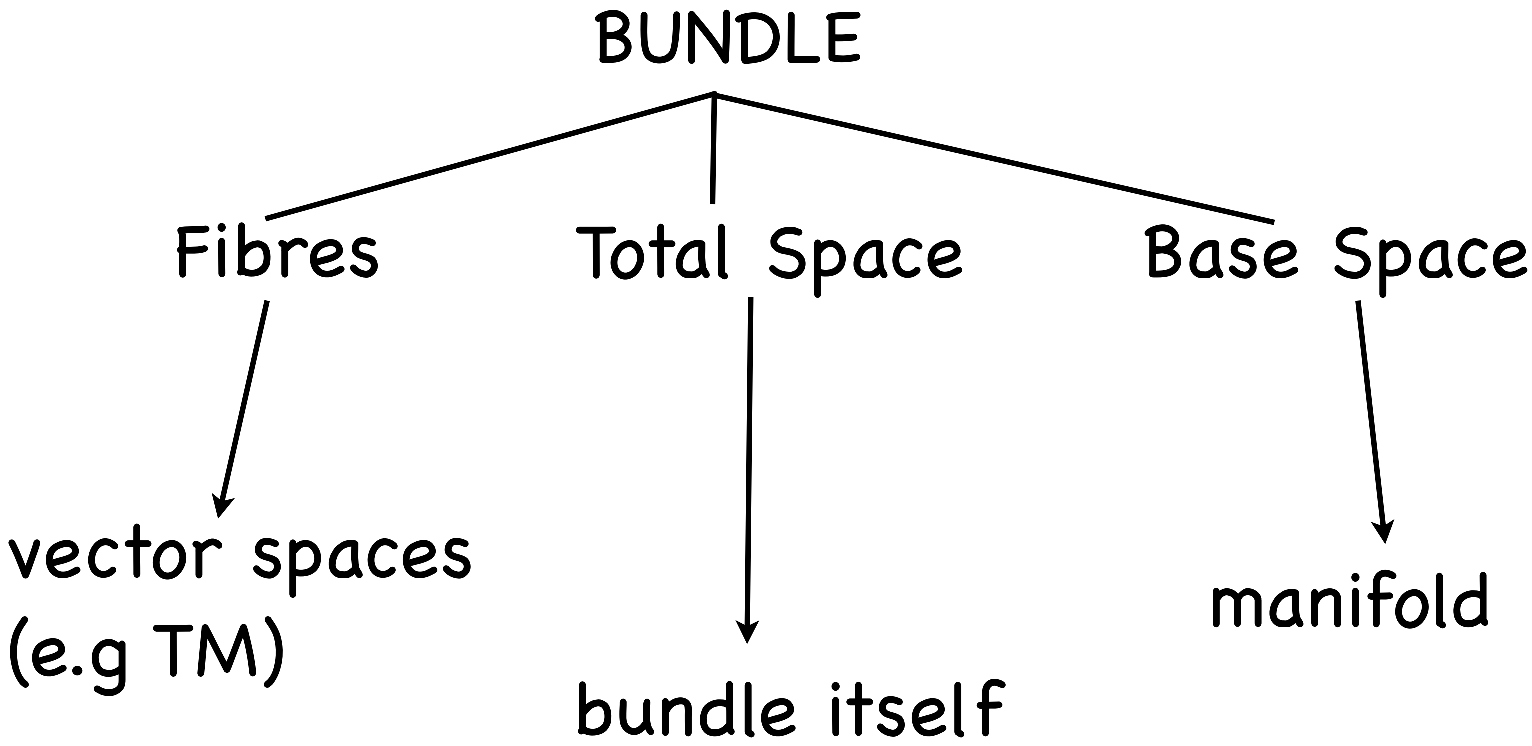
$$x^1, x^2, \ldots, x^p, \theta^1, \theta^2, \ldots, \theta^q.$$

where the x^i are the usual commuting coordinates and θ^j are the grassmann coordinates, s.t.,

$$\theta^j\, \theta^k + \theta^k\, \theta^j = 0.$$

Their nilpotency facilitates us to develop the cohomological/topological field theory.

The local behavior of differential functions on $\mathscr{M}$ is reflected by a 'sheaf'. The sections of this sheaf are referred to as 'superfields'.



<u>Section</u>: A vector field is a section of its tangent bundle.

<u>(Co)Homology Class</u>: The (co)homology class measures the extent to which the bundle is "twisted" - particularly, whether it possesses sections or not.

N=2 supersymmetric sigma Model

Consider a map $\Phi : \Sigma \rightarrow X$, where Σ is the worldsheet, X, called the target space is interpreted as the physical space-time.

The action for such theory is:

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2 z \left[\frac{1}{2} g_{\mu\nu}(X) \partial_z X^\mu \partial_{\bar{z}} X^\nu + g_{\mu\nu} \left(\psi^\mu D_{\bar{z}} \psi^\nu + \lambda^\mu D_z \lambda^\nu \right) + \frac{1}{4} R_{\mu\nu\rho\sigma} \psi^\mu \psi^\nu \lambda^\rho \lambda^\sigma \right]$$

where ψ's are left moving fermions and λ's are right moving fermions, $g_{\mu\nu}$ is the metric on the target manifold and $R_{\mu\nu\rho\sigma}$ is its Riemann tensor.

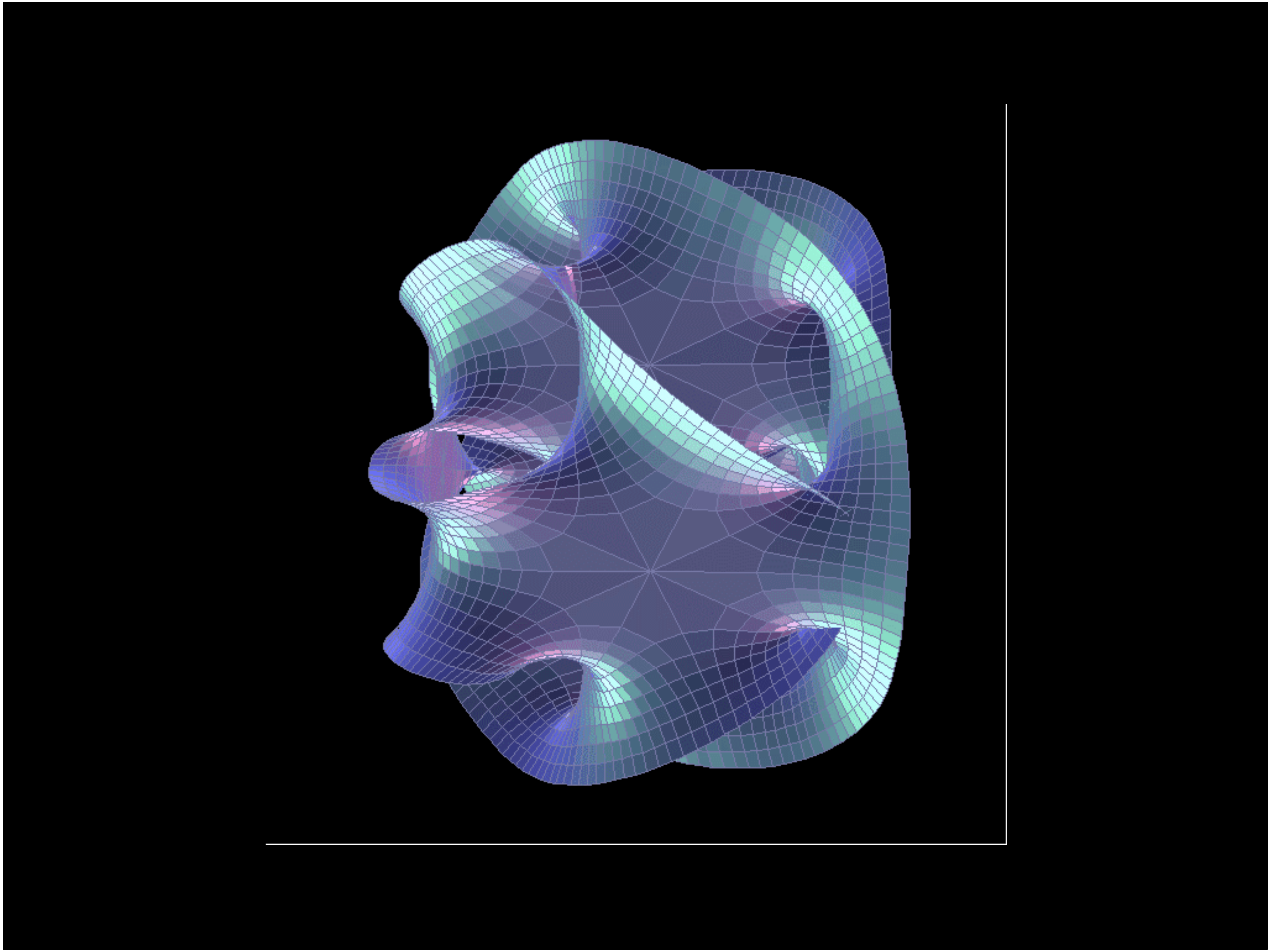
N=2 supersymmetric sigma Model

Kähler metric

complex structure ?
(shown in [13])

Complex Manifold: If we demand that the transition functions $\phi_j \, o \, \phi_i^{-1}$ satisfy the Cauchy-Riemann equations, then the analytic properties of a function f can be studied using its coordinate representation $f \, o \, \phi_i^{-1}$ with assurance that the conclusions drawn are patch independent.

Ricci flat i.e., $R_{i\bar{j}} = 0.$



Topological String Theory

The A Model of Topological String Theory:

$N=2$ means we have two complex supercharges
Q_- and $\overline{Q_+}$.
We will connect them to cohomology because cohomology groups are finite-dimensional.

Dolbeault cohomology ∂ and $\overline{\partial}$.

$Q_A = Q_- + \overline{Q_+}$ can be identified with the deRham operator $d = \partial + \overline{\partial}$, because $d^2 = 0 = Q^2$.

This procedure leads to a "topological twist".
The A-Model is metric independent.

Edward Witten shared the 1990 Fields Medal for his pioneering work in cohomological field theory.

Gromov-Witten invariants

Given an alegebraic variety X, we fix a homology class $\beta=H(X)$ and cycles $Z_1=G(Z_0)$, $Z_2=G(Z_1),\ldots,Z_0 = G(Z_{n-1})$ on X for a map G. Then we consider the following set of curves:

$C\subset X$ of genus g, homology class β, such that $C\cap Z_i \neq\varnothing$ for all $i=0,\ldots,n$.

Gromov-Witten invariants

Let $\alpha_i \in H^*(X)$ be the cohomology class dual to the cycle Z_i, and $\overline{M}_{g,n}$ be Deligne-Mumford moduli space, then

$$\langle I_{g,n,b}\rangle(\alpha_1, \alpha_2,\ldots, \alpha_n) = \int_{\overline{M}_{g,n}} I_{g,n,b}\,(\alpha_1, \alpha_2,\ldots, \alpha_n)$$

is the Gromov Witten invariant which counts the number of rational curves C.

Conclusion

The Gromov-Witten invariants are by design to be unchanged by deformations of the complex structure of X.

Thus they are a complete manifestation of the A-Model's independence of the complex structure of the moduli space.

Gromov-Witten invariants are directly related to another important invariant called Donaldson-Thomson invariant.

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Thank You